The S-Hamiltonian Cycle Problem

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Introduction

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Hamiltonian Cycle: A cycle that visits every vertex exactly once

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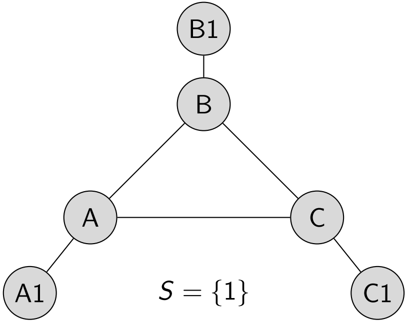

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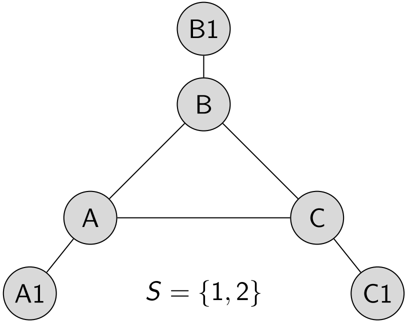

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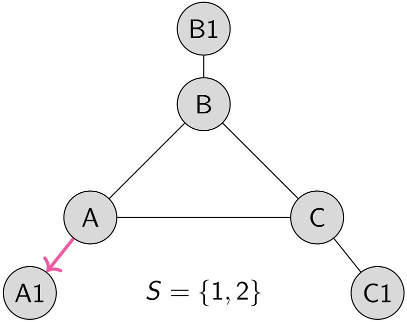

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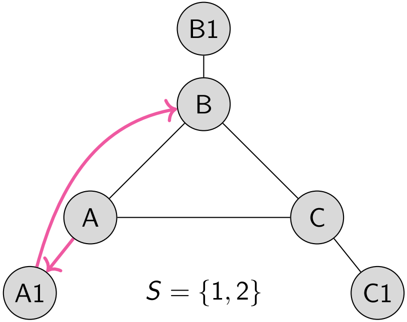

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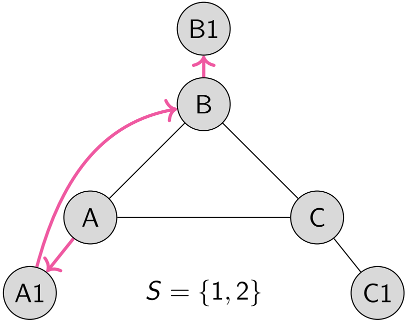

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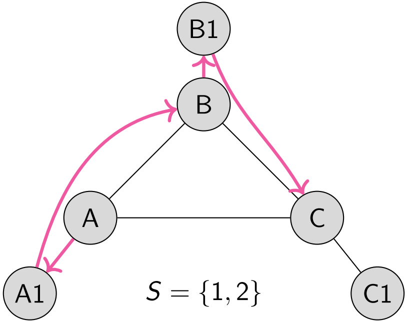

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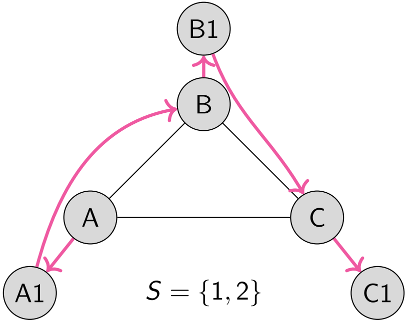

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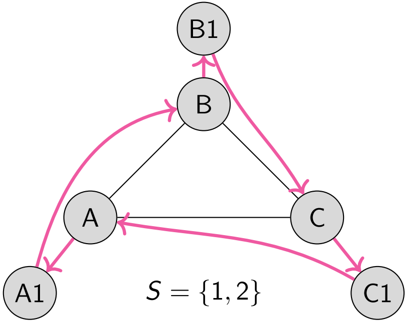

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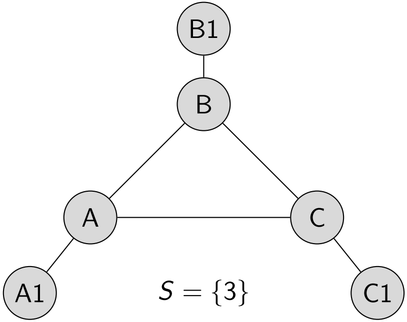

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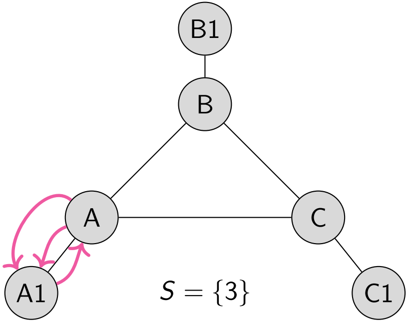

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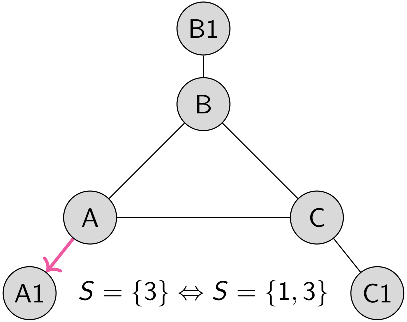

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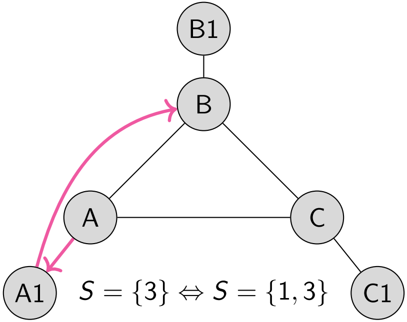

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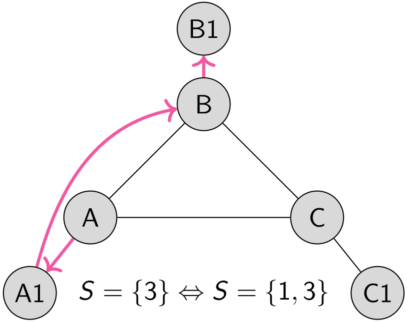

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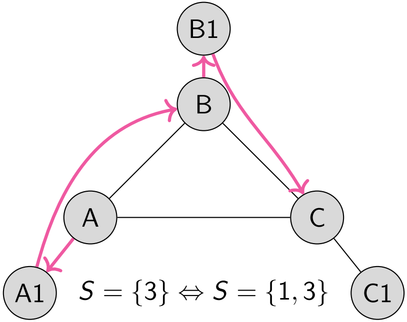

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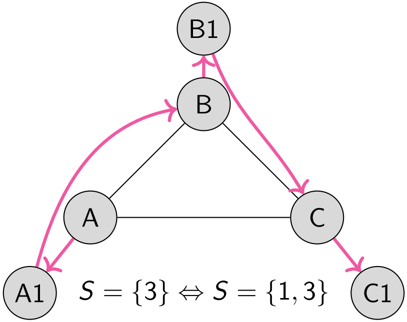

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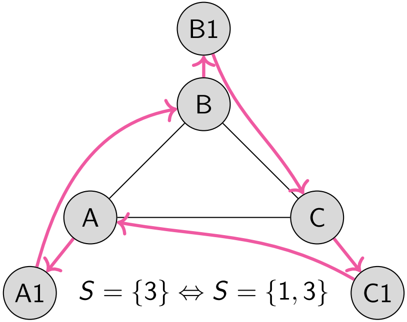

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Known Results

The S-Hamiltonian cycle definition generalizes results from previous papers:

Theorem [P. Underground, 1978]

The $\{1, 2\}$-Hamiltonian cycle problem is NP-complete

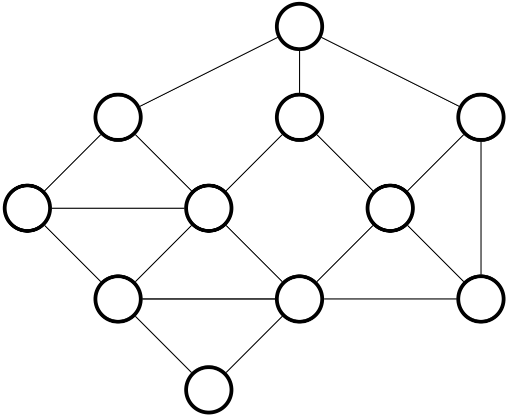
Theorem [J. Karaganis, 1968]

The $\{1, 2, 3\}$-Hamiltonian cycle problem is trivial
(every graph contains such a cycle)

Even-odd trick ($S = \{1, 2, 3\}$)

Theorem [J. Karaganis, 1978]

Every connected graph admits a $\{1, 2, 3\}$-Hamiltonian cycle



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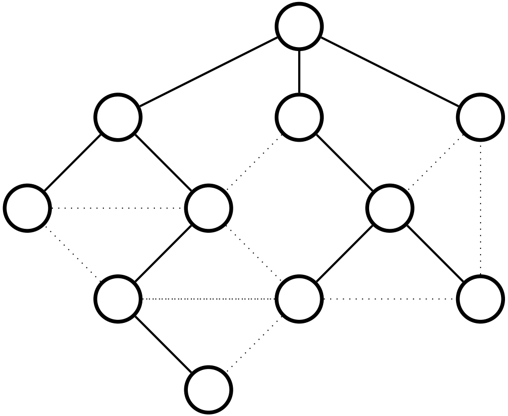

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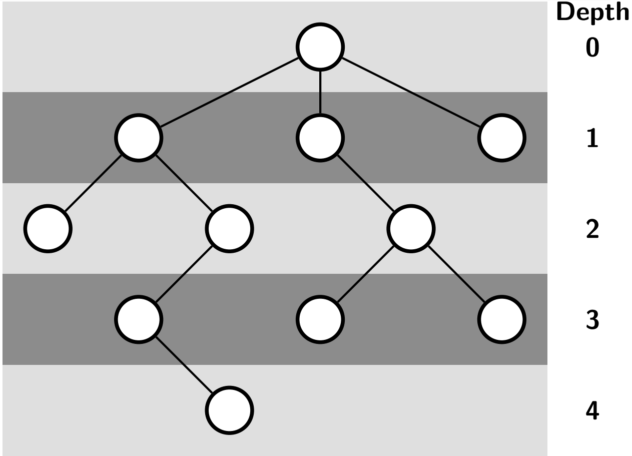

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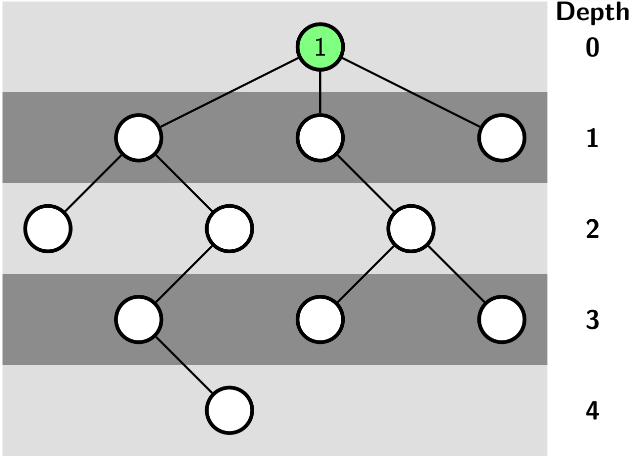

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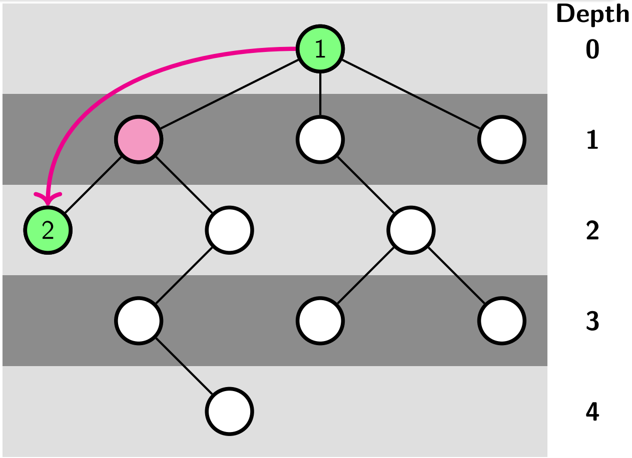

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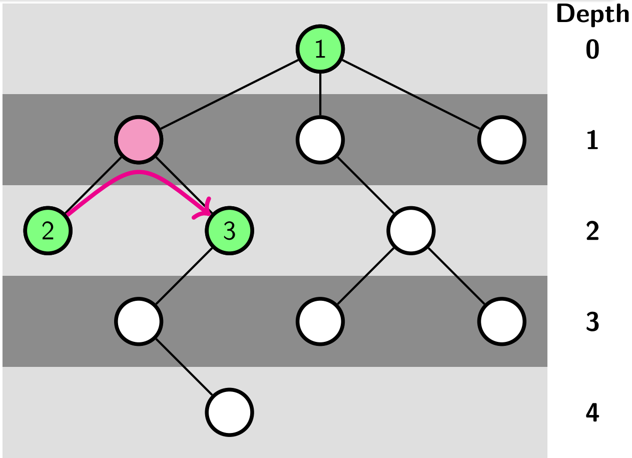

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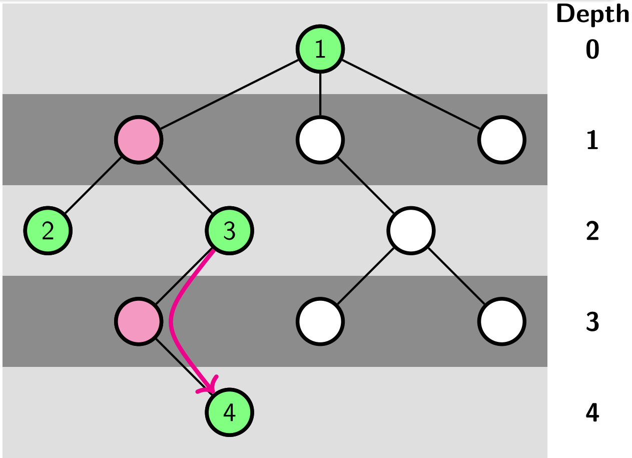

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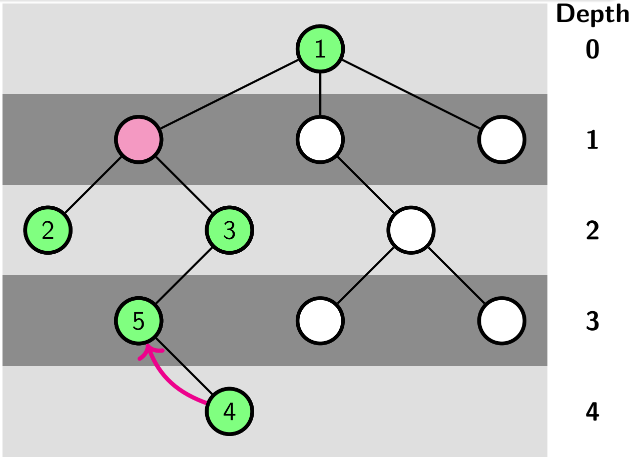

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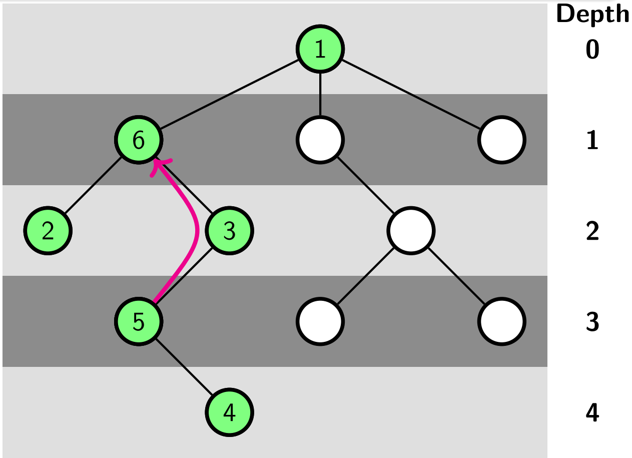

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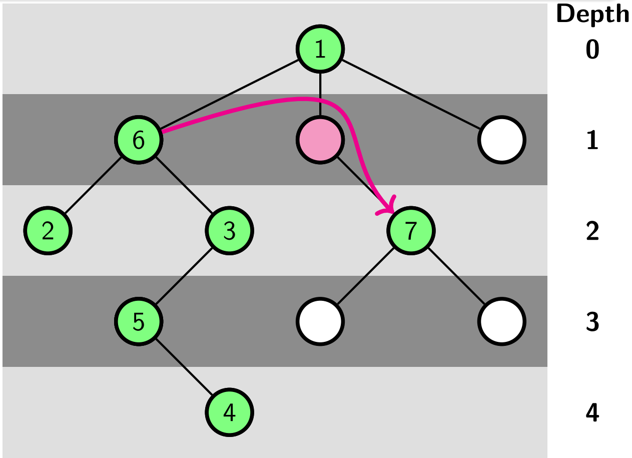

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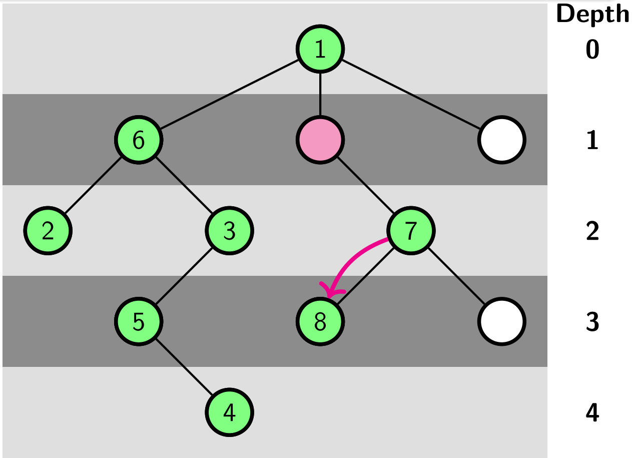

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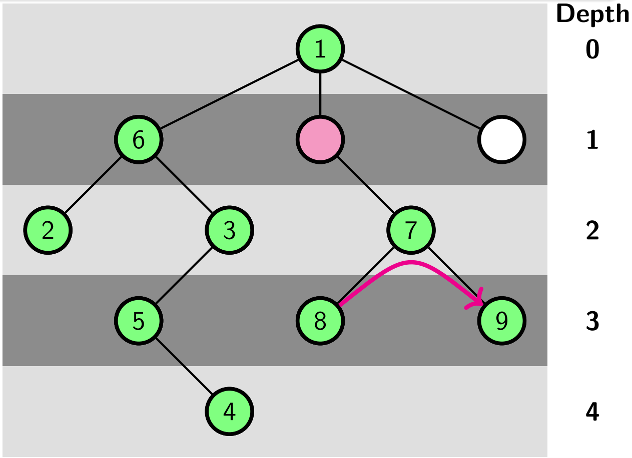

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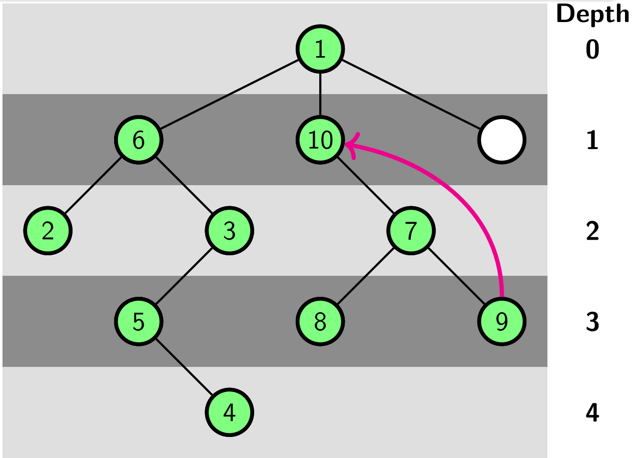

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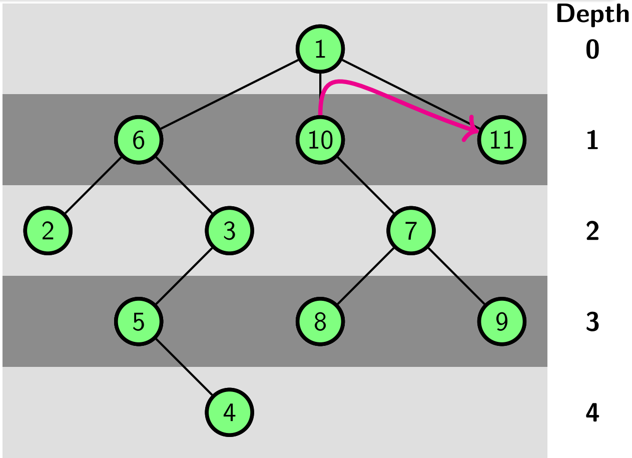

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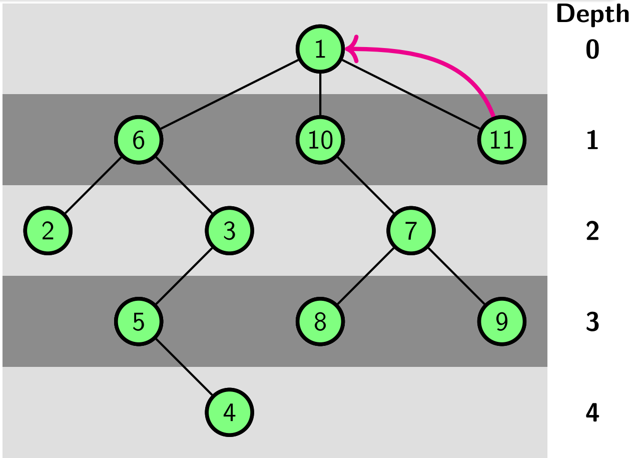

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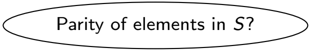

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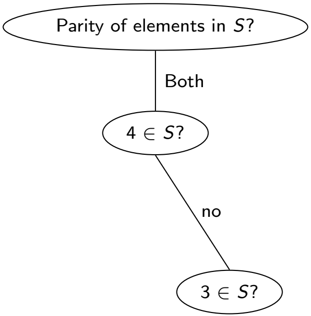

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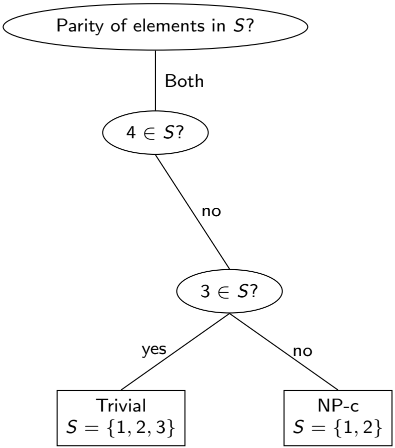

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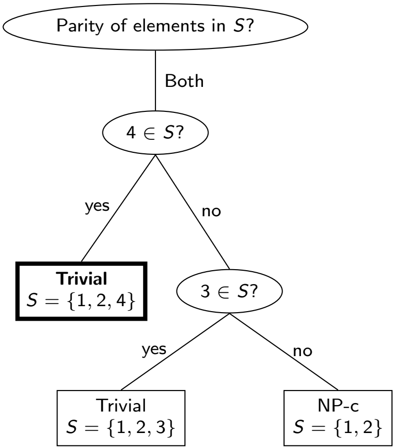

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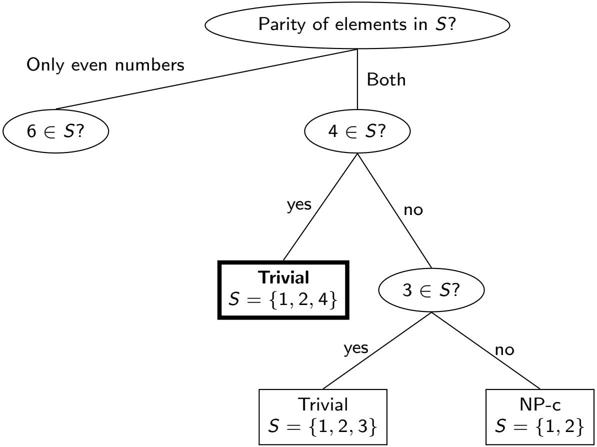

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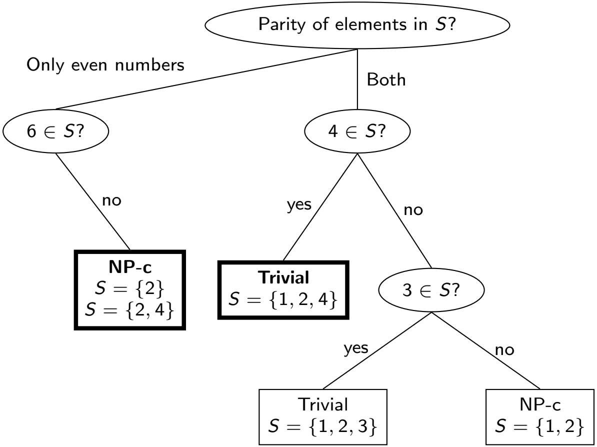

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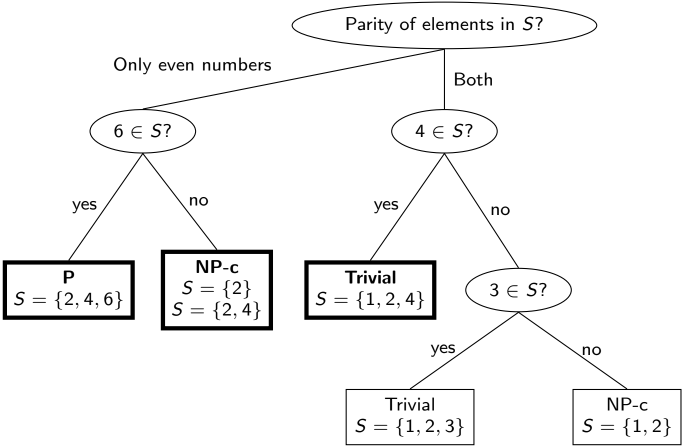

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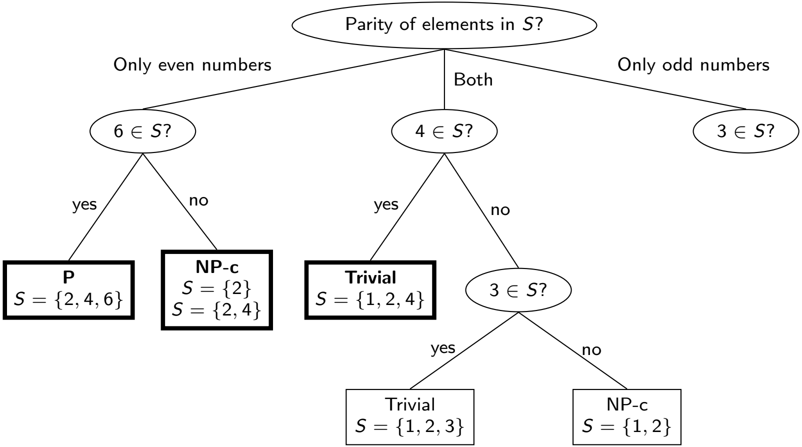

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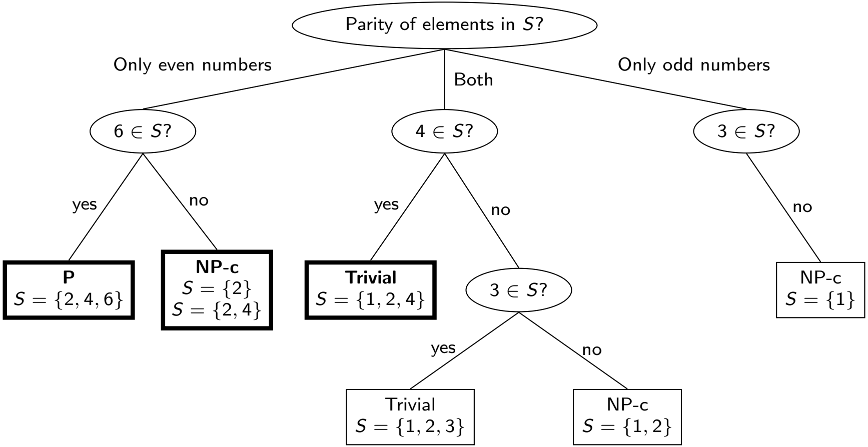

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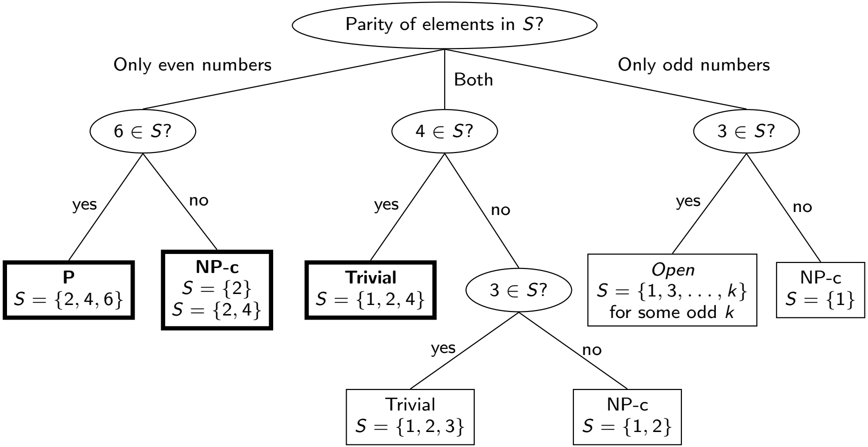

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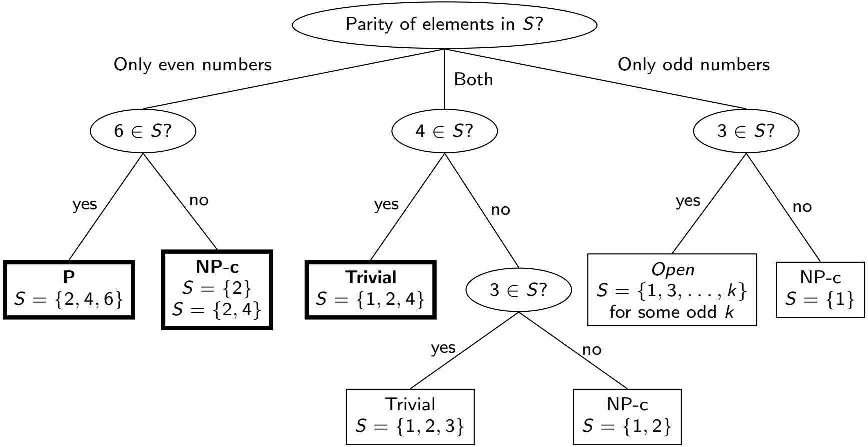

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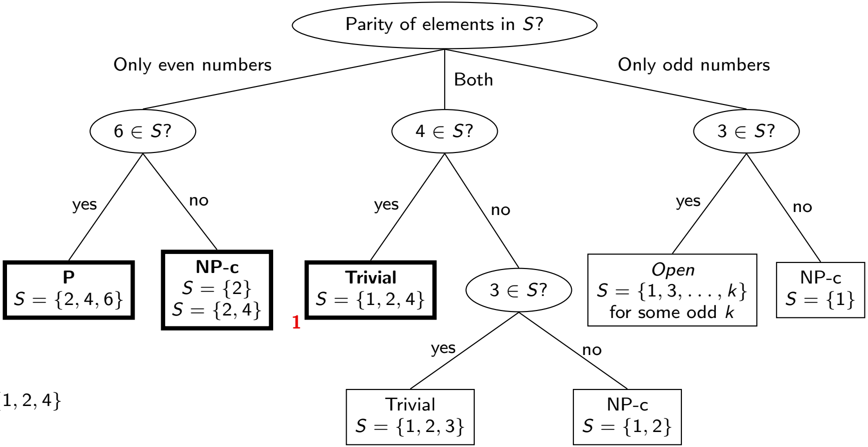
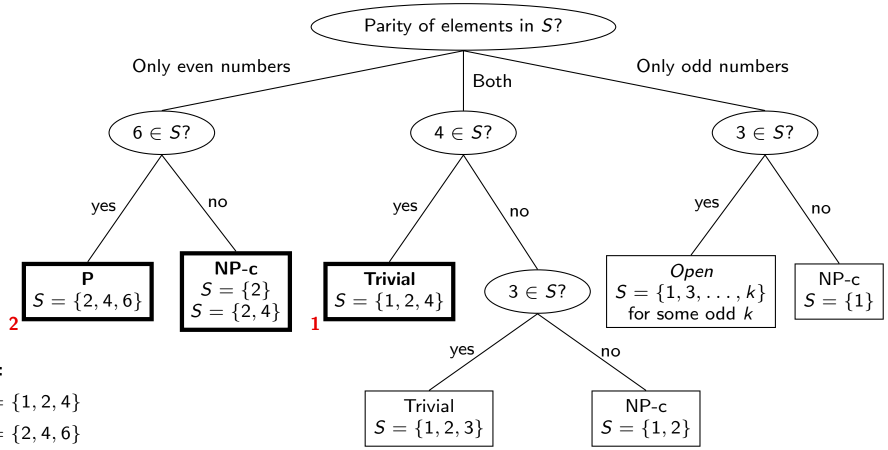


Roadmap:

1. $S = \{1, 2, 4\}$

Our results

A decision tree for the complexity of the S-Hamiltonian cycle problem



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1. $S = \{1, 2, 4\}$
2. $S = \{2, 4, 6\}$

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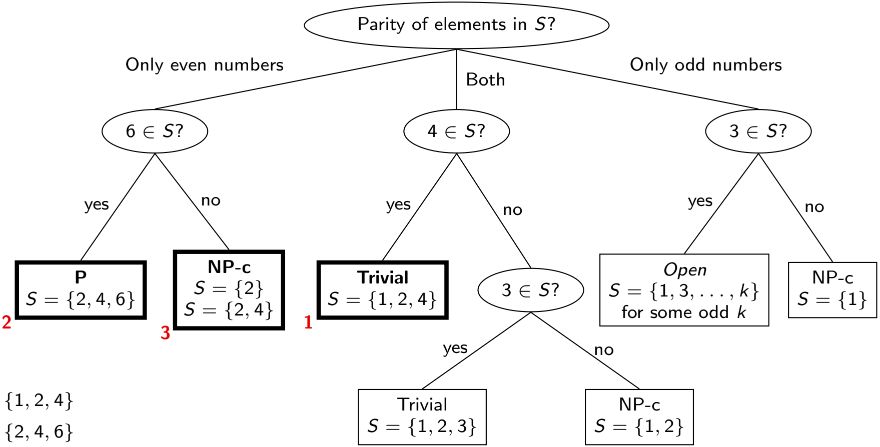

Roadmap:

1. $S = \{1, 2, 4\}$
2. $S = \{2, 4, 6\}$
3. $S = \{2\}$

$S = \{1, 2, 4\}$ is trivial

Theorem [This paper]

Every graph admits a $\{1, 2, 4\}$-Hamiltonian cycle

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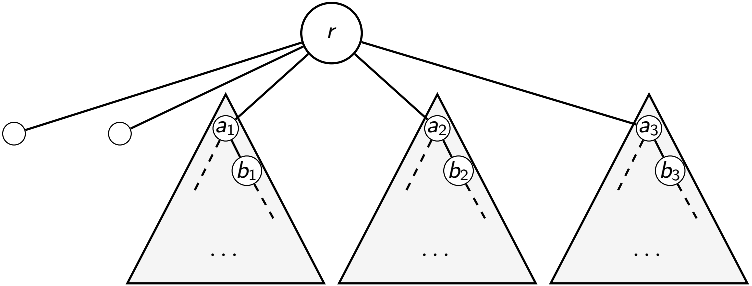

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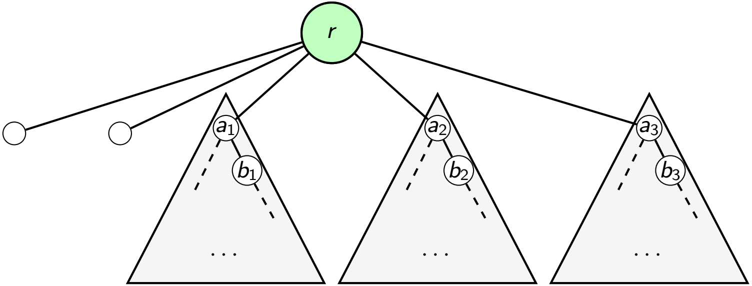

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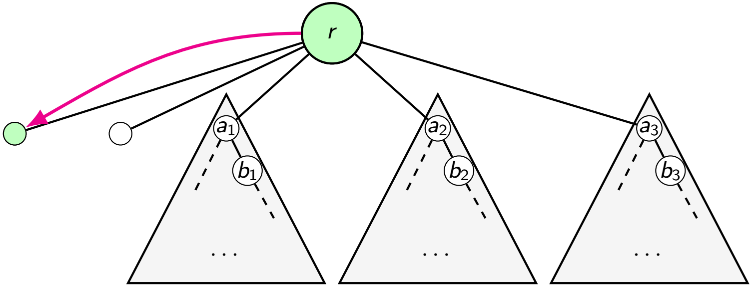

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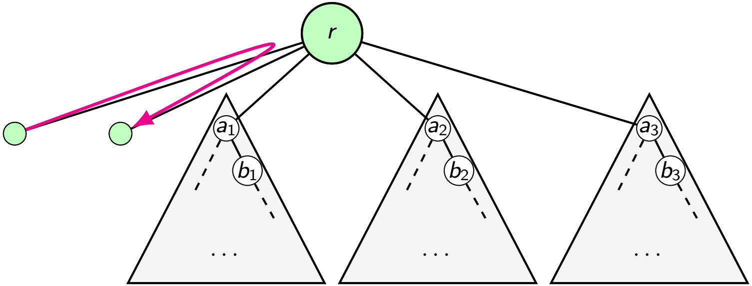

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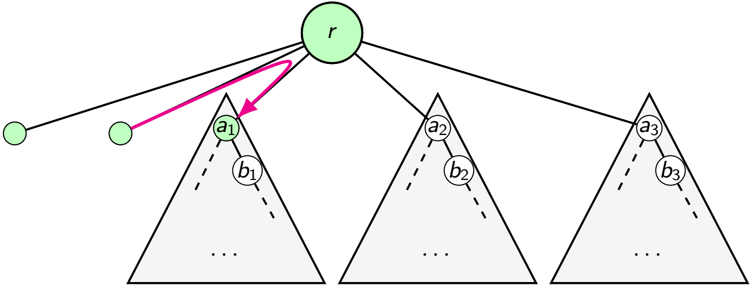

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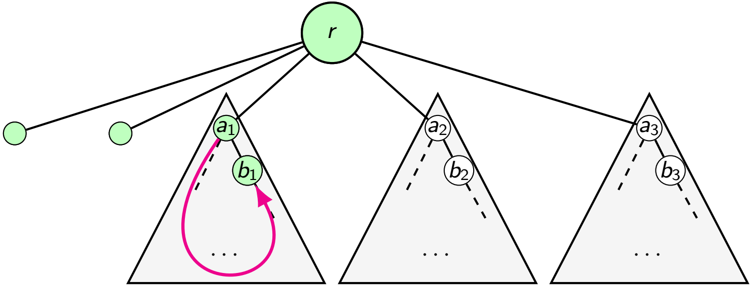

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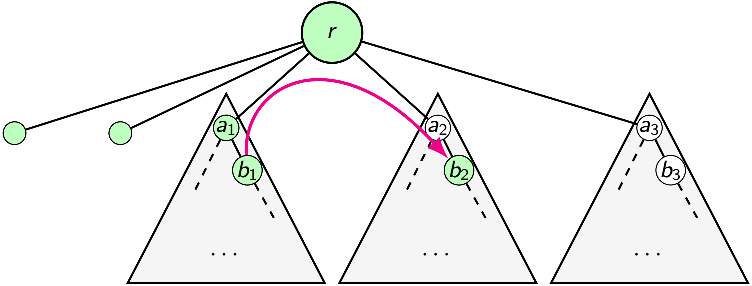

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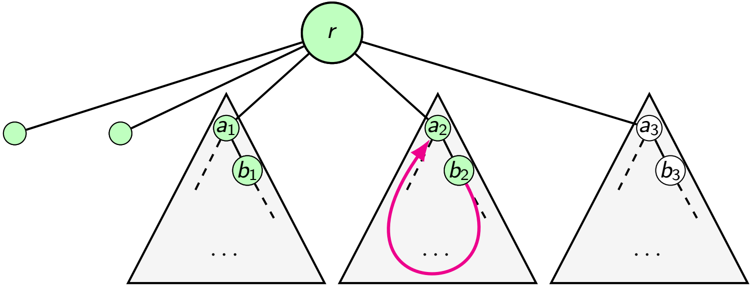

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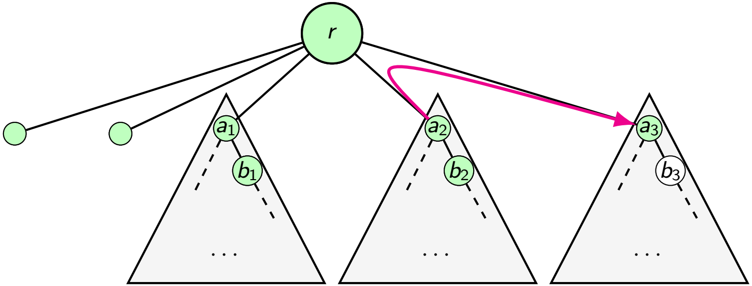

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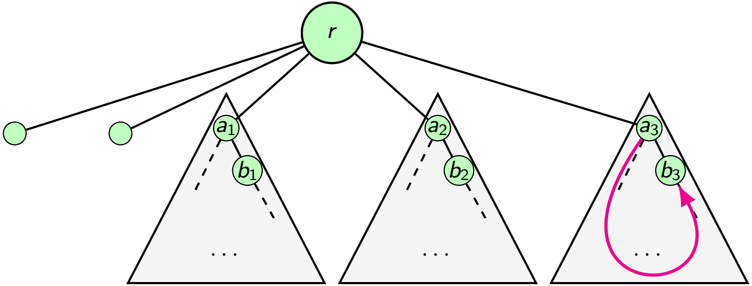

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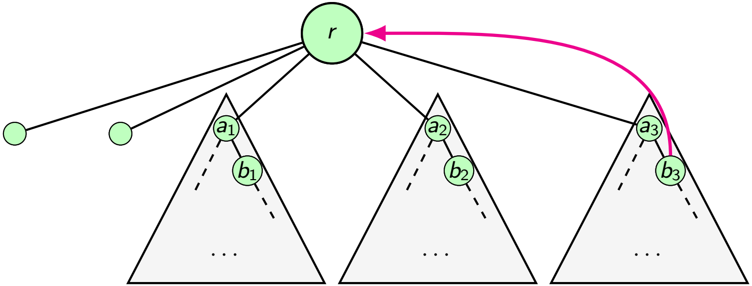

$S = \{2, 4, 6\}$ is polynomial

Theorem [This paper]

The $\{2, 4, 6\}$-Hamiltonian cycle problem can be solved in **polynomial time**

A graph admits a $\{2, 4, 6\}$-Hamiltonian cycle
if and only if the graph is **not bipartite**

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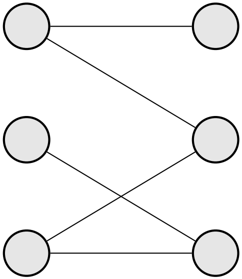

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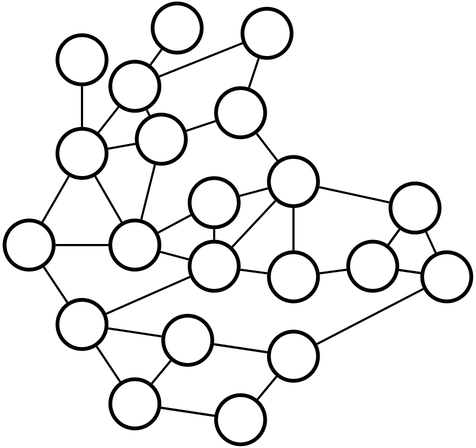
$S = \{2, 4, 6\}$ on a non-bipartite graph

Theorem [This paper]

Every non-bipartite graph has a $\{2, 4, 6\}$-Hamiltonian cycle



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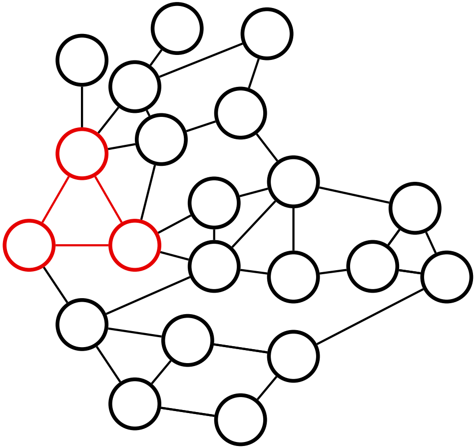

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Every non-bipartite graph has a $\{2, 4, 6\}$-Hamiltonian cycle

- Find an **odd length cycle**
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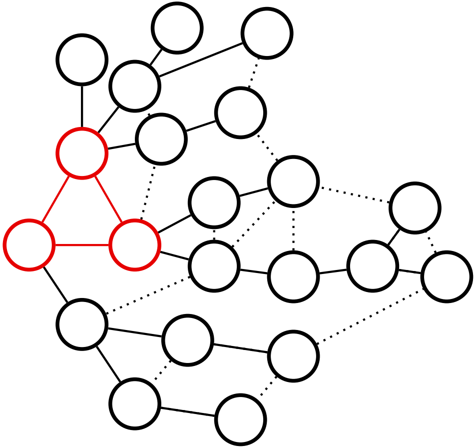

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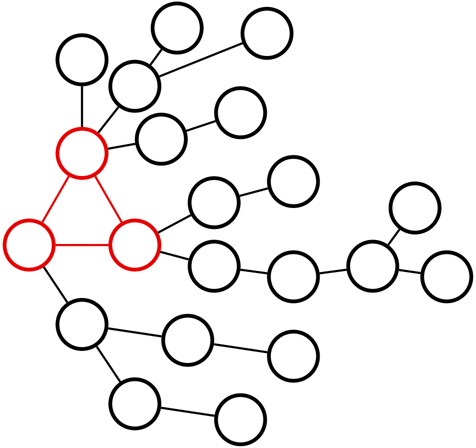

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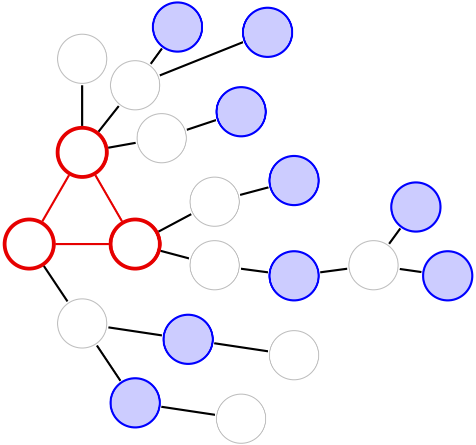

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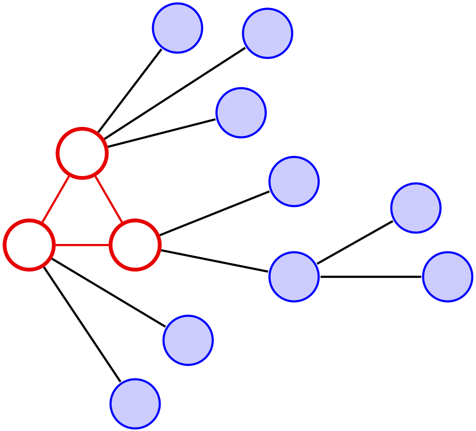

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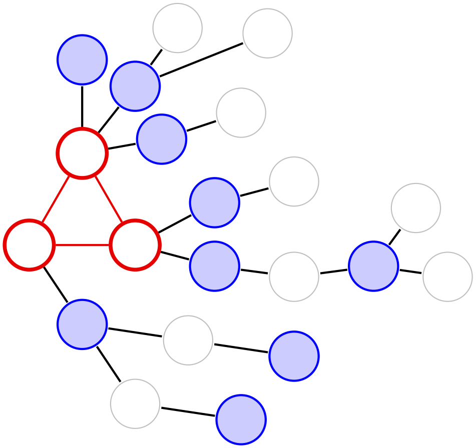

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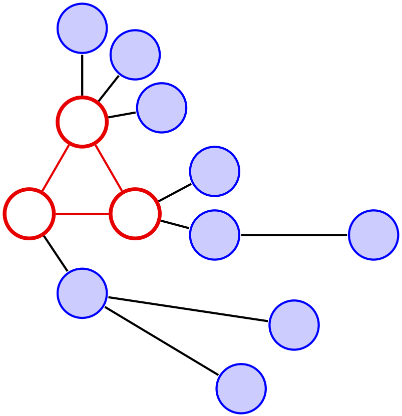

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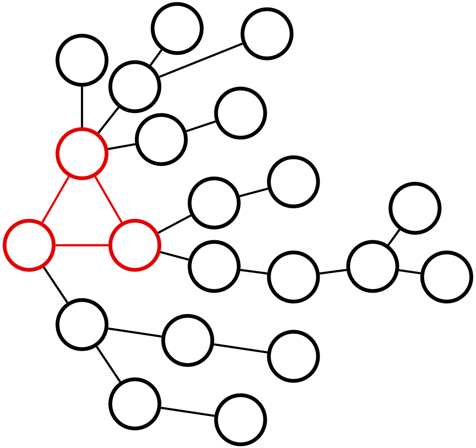
- Same on vertices at odd depth
- Traverse the red cycle twice, alternating odd-depth and even-depth cycles of the subtrees



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Theorem [This paper]

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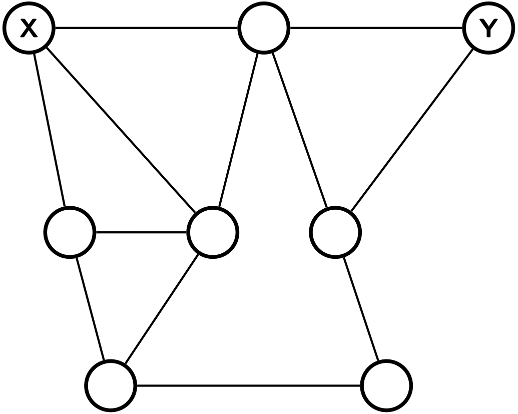

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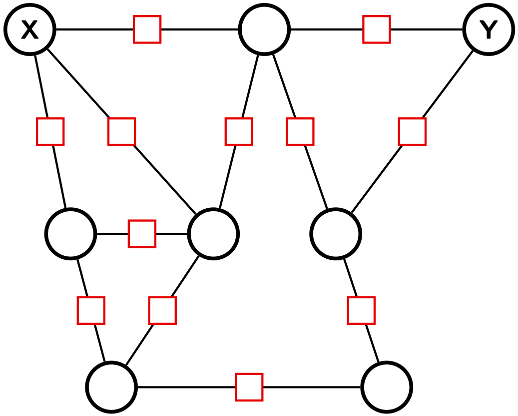

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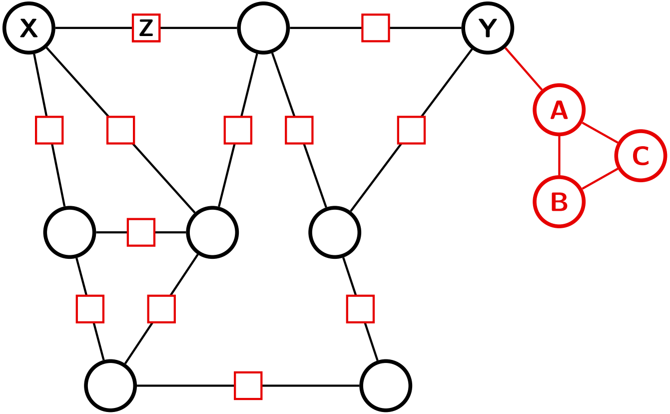

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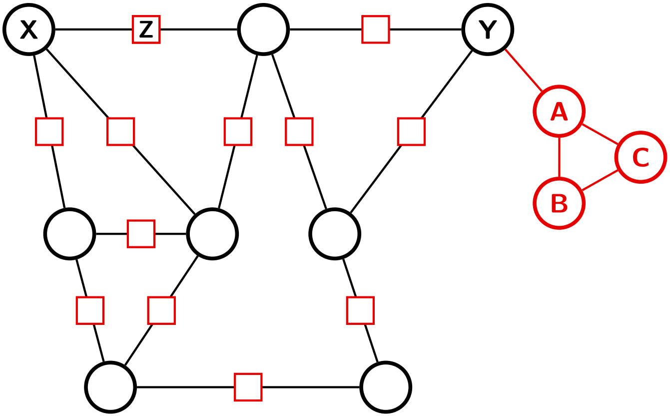

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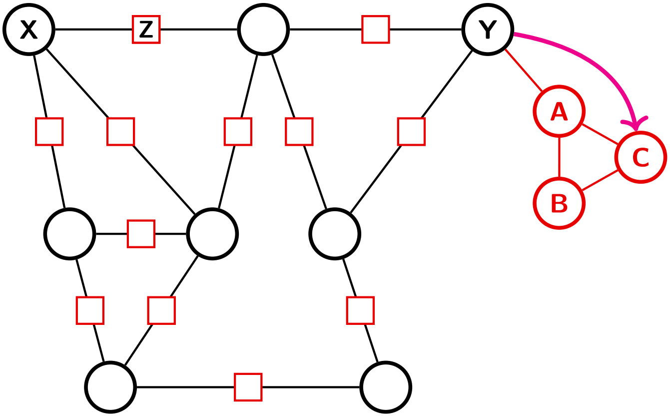

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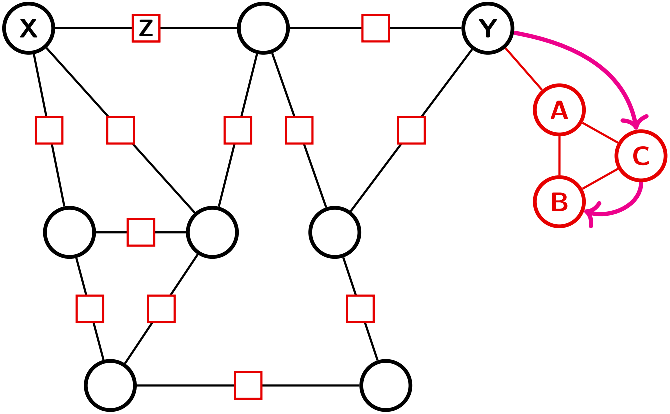

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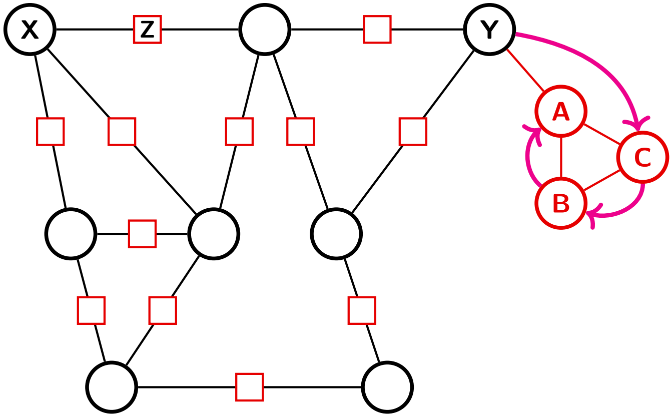

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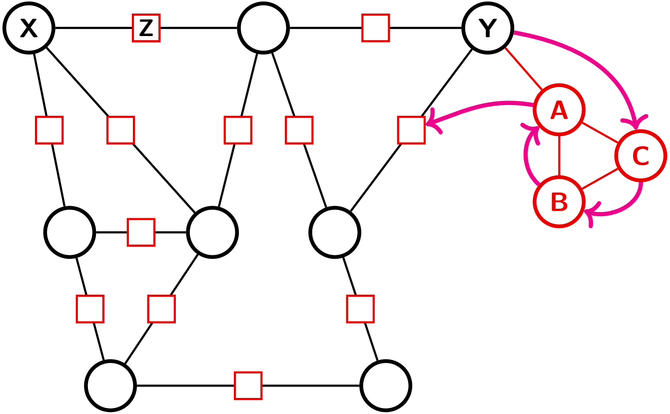

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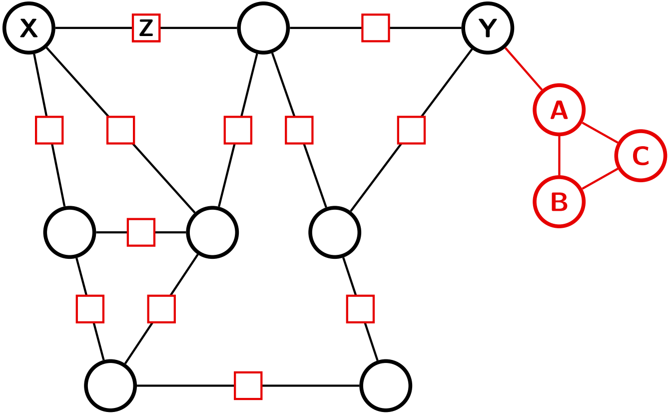
- The triangle is the only way to alternate between circles and squares

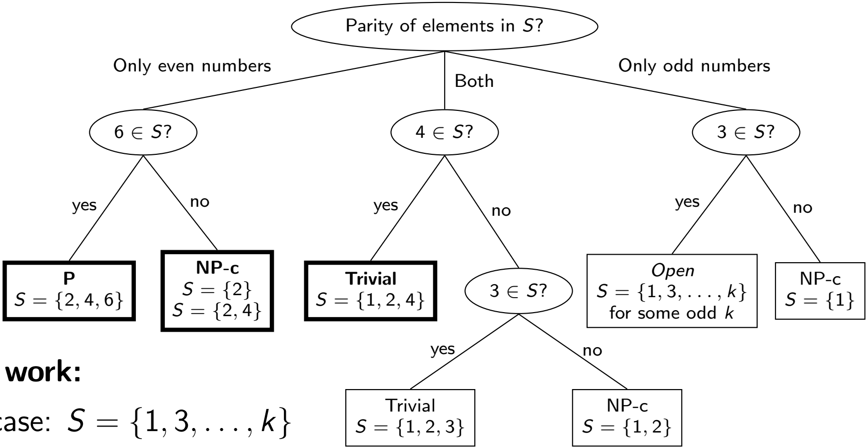


Conclusion

Future work:

Open case: $S = \{1, 3, \ldots, k\}$

Directed Graphs



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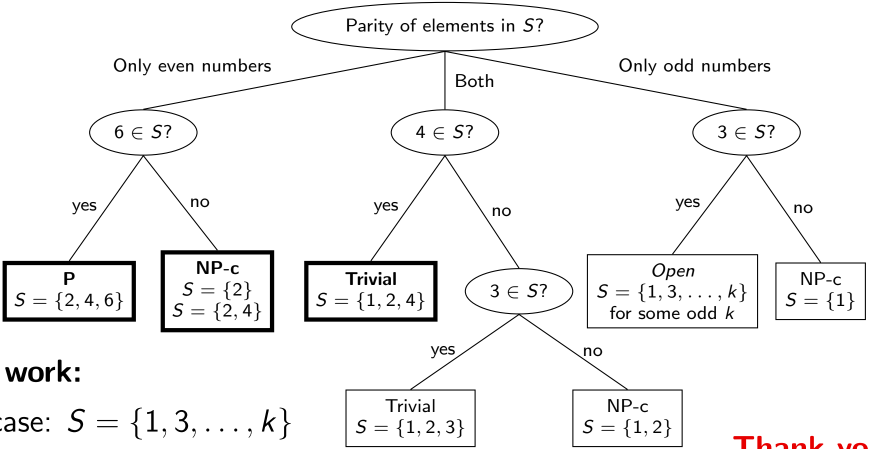

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Thank you!